1-3 Real Numbers and the Number Line

Common Core State Standards

Prepares for N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

MP 1, MP 3, MP 6

Objectives To classify, graph, and compare real numbers
To find and estimate square roots

This problem involves a special group of numbers.

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

If the pattern continues, which will be the first figure to contain more than 200 square units? Explain your reasoning.

1^2 2^2 3^2 4^2 5^2 . . . 14^2, 15^2

1 4 9 16 25 . . . 196, 225

The diagrams in the Solve It model what happens when you multiply a number by itself to form a product. When you do this, the original number is called a *square root* of the product.

take note **Key Concept** Square Root

Algebra A number a is a **square root** of a number b if $a^2 = b$.

Example $7^2 = 49$, so 7 is a square root of 49.

Lesson Vocabulary
- square root
- radicand
- radical
- perfect square
- set
- element of a set
- subset
- rational numbers
- natural numbers
- whole numbers
- integers
- irrational numbers
- real numbers
- inequality

$\underbrace{\sqrt{\text{radicand}}}_{\text{radical}}$

radical symbol

$\sqrt{16} = 4$

$\sqrt[3]{8} = 2$

Alg II

...k

...an you find a square root?
Find a number that you can multiply by itself to get a product that is equal to the radicand.

Problem 1 Simplifying Square Root Expressions

What is the simplified form of each expression?

A $\sqrt{81} = 9$

B $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

Got It? 1. What is the simplified form of each expression?

a. $\sqrt{64}$ 8 b. $\sqrt{25}$ 5 c. $\sqrt{\frac{1}{36}}$ $\frac{1}{6}$ d. $\sqrt{\frac{81}{121}} = \frac{9}{11}$

Check $\left(\frac{9}{11}\right)\left(\frac{9}{11}\right) = \frac{81}{121}$ ✓

The square of an integer is called a **perfect square**. For example, 49 is a perfect square because $7^2 = 49$. When a radicand is not a perfect square, you can estimate the square root of the radicand.

Problem 2 Estimating a Square Root STEM

Biology Lobster eyes are made of tiny square regions. Under a microscope, the surface of the eye looks like graph paper. A scientist measures the area of one of the squares to be 386 square microns. What is the approximate side length of the square to the nearest micron?

Plan
...n you get ...?
The square root of the area of a square is equal to its side length. So, find $\sqrt{386}$.

$\sqrt{361} \xleftarrow{25} \sqrt{386} \xrightarrow{14} \sqrt{400}$

19 20

$\sqrt{25} \xleftarrow{9} \sqrt{34} \xrightarrow{2} \sqrt{36}$

5 6 5.83...

$\sqrt{64} \xleftarrow{6} \sqrt{70} \xrightarrow{11} \sqrt{81}$

8 9

Got It? 2. What is the value of $\sqrt{34}$ to the nearest integer?

Essential Understanding Numbers can be classified by their characteristics. Some types of numbers can be represented on the number line.

You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object is called an **element of the set.** A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

{ } { }

Natural Numbers : {1, 2, 3, 4, ...}

Whole Numbers : {0, 1, 2, 3, 4, ...}

Integers : {..., -3, -2, -1, 0, 1, 2, 3, ...}

Rational Numbers - any number that can be written as a fraction using

REAL

integers.
[decimals either terminate or repeat]

NUMBERS

Irrational Numbers – any number that cannot be written as a fraction using integers
[decimals never terminate and never repeat]
π, $\sqrt{10}$, e

~~$i = \sqrt{-1}$~~
~~$\sqrt{-16} = \sqrt{16}\sqrt{-1}$~~
~~$4i$~~

Think

What clues can you use to classify real numbers?
Look for negative signs, fractions, decimals that do or do not terminate or repeat, and radicands [that are] not perfect [squares].

Problem 3 Classifying Real Numbers

To which subsets of the real numbers does each number belong?

A 15: rational, integer, whole, natural $\frac{15}{1}$

B −1.4583 rational $\frac{-14583}{10000}$

C $\sqrt{57}$ irrational

Got It? **3.** To which subsets of the real numbers does each number belong?

a. $\sqrt{9}$ **b.** $\frac{3}{10}$ **c.** −0.45 **d.** $\sqrt{12}$

take note

Concept Summary Real Numbers

Real Numbers

Rational Numbers	Integers	Whole Numbers	Natural Numbers	Irrational Numbers
$-\frac{2}{3}$, $0.\overline{3}$, $\sqrt{0.25}$	-3, $-\frac{10}{5}$, $-\sqrt{16}$	0	$\sqrt{25}$, $\frac{4}{2}$, 7	$\sqrt{10}$, $-\sqrt{123}$, 0.1010010001…, π

An **inequality** is a mathematical sentence that compares the values of two expressions using an inequality symbol. The symbols are

$<$, less than	$\leq$, less than or equal to
$>$, greater than	$\geq$, greater than or equal to

Plan

...n you ...e numbers?
Write the numbers in the same form, such as decimal form.

Problem 4 Comparing Real Numbers

What is an inequality that compares the numbers $\sqrt{17}$ and $4\frac{1}{3}$?

Got It? **4. a.** What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?

b. Reasoning In Problem 4, is there another inequality you can write that compares the two numbers? Explain.

You can graph and order all real numbers using a number line.

Problem 5 Graphing and Ordering Real Numbers

Multiple Choice What is the order of $\sqrt{4}$, 0.4, $-\frac{2}{3}$, $\sqrt{2}$, and -1.5 from least to greatest?

(A) $-\frac{2}{3}, 0.4, -1.5, \sqrt{2}, \sqrt{4}$	(C) $-1.5, -\frac{2}{3}, 0.4, \sqrt{2}, \sqrt{4}$
(B) $-1.5, \sqrt{2}, 0.4, \sqrt{4}, -\frac{2}{3}$	(D) $\sqrt{4}, \sqrt{2}, 0.4, -\frac{2}{3}, -1.5$

Think

Why is it useful to rewrite numbers in decimal form?
It allows you to compare numbers whose values are close, like $\frac{1}{4}$ and 0.26.

Got It? **5.** Graph 3.5, -2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?

Lesson Check

Do you know HOW?

Name the subset(s) of the real numbers to which each number belongs.

1. $\sqrt{11}$ **2.** -7

3. Order $\frac{47}{10}$, 4.1, -5, and $\sqrt{16}$ from least to greatest.

4. A square card has an area of 15 in.^{2}. What is the approximate side length of the card?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary What are the two subsets of the real numbers that form the set of real numbers?

6. Vocabulary Give an example of a rational number that is not an integer.

Reasoning **Tell whether each square root is *rational* or *irrational*. Explain.**

7. $\sqrt{100}$ **8.** $\sqrt{0.29}$

1-3 Real Numbers and the Number Line

Vocabulary

$\sqrt{289} = 17$ $\quad$ $\sqrt{20} = 4. \ . \ . \ .$

Review

1. Circle the numbers that are *perfect squares*.

1 $\quad$ 100 $\quad$ ~~12~~ $\quad$ 121 $\quad$ 16 $\quad$ ~~200~~ $\quad$ ~~20~~ $\quad$ 289

Vocabulary Builder

square root (noun) **skwer root**

Definition: The **square root** of a number is a number that when multiplied by itself is equal to the given number.

Using Symbols: $\sqrt{16} = 4$

Using Words: The **square root** of 16 is 4. It means, "I multiply 4 by itself to get 16."

square root

$\sqrt{16} = 4$

because

$4^2 = 16$

Use Your Vocabulary

2. Use what you know about *perfect squares* and *square roots* to complete the table.

Number	Number Squared
1	1
2	4
3	9
4	16
5	25
6	36

Number	Number Squared
7	49
8	64
9	81
10	100
11	121
12	144

n	n^2
13	169
14	196
15	225
20	400
25	625
30	900
40	1600
100	10000

Problem 1 Simplifying Square Root Expressions

Got It? **What is the simplified form of $\sqrt{64}$?**

3. Circle the equation that uses the positive square root of 64.

$16 \cdot 4 = 64$	$32 \cdot 2 = 64$	$8 \cdot 8 = 64$

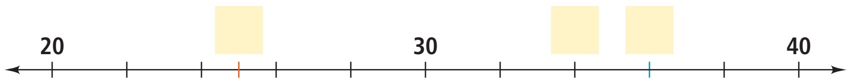
4. The simplified form of $\sqrt{64}$ is ___.

Problem 2 Estimating a Square Root

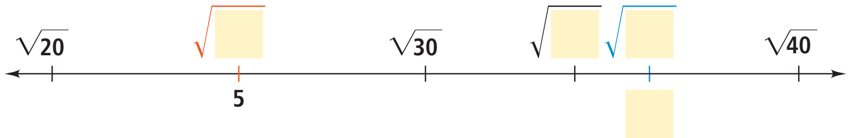
Got It? **What is the value of $\sqrt{34}$ to the nearest integer?**

5. Use the number lines below to find the perfect squares closest to 34.

Write 25, 34, and 36 in the correct positions on the number line.



Complete the number line with square roots.



6. Since 34 is closer to ___ than to ___,

$\sqrt{34}$ is closer to ___ than to ___.

So, the value of $\sqrt{34}$ to the nearest integer is ___.

You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object in the set is called an **element** of the set. A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

7. Complete the *sets* of numbers.

Natural numbers $\{1, \_\_\_, 3, \ldots\}$

Whole numbers $\{\_\_\_, 1, \_\_\_, 3, \ldots\}$

Integers $\{\ldots, -2, \_\_\_, 0, 1, \_\_\_, 3, \ldots\}$

A **rational number** is any number that you can write in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A rational number in decimal form is either a terminating decimal such as 5.45 or a repeating decimal such as 0.333..., which you can write as $0.\overline{3}$.

8. Cross out the numbers that are NOT *rational numbers*.

π	$-\frac{7}{4}$	$\sqrt{5}$	$0.\overline{9}$	7.35

An **irrational number** cannot be represented as the quotient of two integers. In decimal form, irrational numbers do not terminate or repeat. Irrational numbers include π and $\sqrt{2}$.

Problem 3 Classifying Real Numbers

Got It? **To which subsets of the real numbers does each number belong?**

$\sqrt{9}$ $\frac{3}{10}$ -0.45 $\sqrt{12}$

9. Is each number an element of the set? Place a ✔ if it is. Place an ✗ if it is not.

Number	Whole Numbers	Integers	Rational Numbers	Irrational Numbers
3 = $\sqrt{9}$	✔	✔	✔	✗
0.3 = $\frac{3}{10}$	✗	✗	✔	✗
-0.45	✗	✗	✔	✗
$\sqrt{12}$	✗	✗	✗	✔

take note

Concept Summary Real Numbers

10. Write each of the numbers -7, -5.43, 0, $\frac{3}{7}$, π, and $\sqrt{7}$ in a box below. The number 5 has been placed for you.

.42857429

Real Numbers

Rational numbers	Integers	Whole numbers	Natural numbers	Irrational numbers
$\frac{3}{7}$ -5.43	-7	0	5	π $\sqrt{7}$

Problem 4 Comparing Real Numbers

$\neq$ Inequalities

$<$ $\leq$

$>$ $\geq$

Got It? **What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?**

11. What is the approximate value of $\sqrt{129}$ to the nearest hundredth?

11.35

12. Use $<$, $>$, or $=$ to complete the statement.

$\sqrt{129}$ $<$ 11.52

11.35
11.52

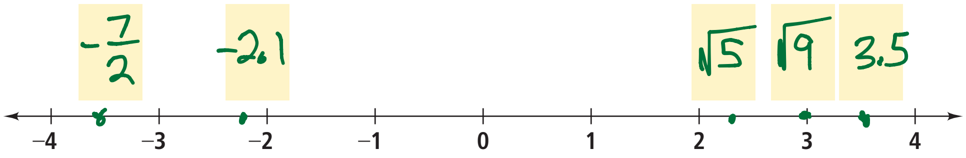
Problem 5 Graphing and Ordering Real Numbers

Got It? **Graph 3.5, -2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?**

13. Simplify the radicals and convert the fraction to a mixed number.

$\sqrt{9} =$ 3 $\qquad -\frac{7}{2} =$ -3.5 $\qquad \sqrt{5} \approx$ 2.23

14. Now use the number line to graph the five original numbers. Be sure to label each point with the correct number.



15. Now list the five original numbers from *least* to *greatest*.

$-\frac{7}{2}$, -2.1, $\sqrt{5}$, $\sqrt{9}$, 3.5

Lesson Check • Do you UNDERSTAND?

Reasoning **Tell whether $\sqrt{100}$ and $\sqrt{0.29}$ are *rational* or *irrational*. Explain.**

16. First try to simplify the expression. If it does not simplify, put an ✗ in the box.

$\sqrt{100} =$ 10 $\qquad \sqrt{0.29} =$ .53...

17. Tell whether each square root is *rational* or *irrational*. Explain your reasoning.

$\sqrt{100}$ is rational; radicand is a perfect square

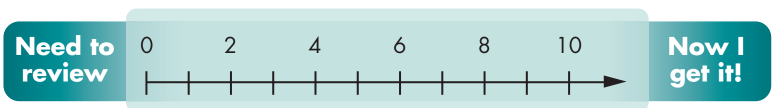
$\sqrt{0.29}$ is irrational; radicand is not a perfect square

Math Success

Check off the vocabulary words that you understand.

☐ square root ☐ rational numbers ☐ irrational numbers ☐ real numbers

Rate how well you can *classify and order real numbers*.



Name________________________________ Class______________ Date______________

1-3 Practice

Simplify each expression.

1. $\sqrt{4}$ $=2$
2. $\sqrt{36}$
3. $\sqrt{25}$ $=5$
4. $\sqrt{81}$ $=9$
5. $\sqrt{121}$
6. $\sqrt{169}$ $=13$
7. $\sqrt{625}$ $=25$
8. $\sqrt{225}$
9. $\sqrt{\frac{64}{9}}$ $=\frac{8}{3}$
10. $\sqrt{\frac{25}{81}}$ $=\frac{5}{9}$
11. $\sqrt{\frac{225}{169}}$
12. $\sqrt{\frac{1}{625}}$ $=\frac{1}{25}$
13. $\sqrt{0.64}$ $=0.8$
14. $\sqrt{0.81}$
15. $\sqrt{6.25}$ $=2.5$

$0.8 \times 0.8 = .64$

$\sqrt{6.4}$ irrational $\sqrt{.0064} = .08$

$\sqrt{.064}$ irrational $\sqrt{1.21} = 1.1$

Estimate the square root. Round to the nearest integer.

16. $\sqrt{10}$ $\sqrt{9} < \sqrt{10}$ $\sqrt{16}$ — 3, 4
17. $\sqrt{15}$
18. $\sqrt{38}$ $\sqrt{36} \leftarrow \sqrt{38}$ $\sqrt{49}$ — 6, 7
19. $\sqrt{50}$ $\sqrt{49} < \sqrt{50}$ $\sqrt{64}$ — 7, 8
20. $\sqrt{16.8}$
21. $\sqrt{37.5}$ $\sqrt{36} \leftarrow \sqrt{37.5}$ $\sqrt{49}$ — 6
22. $\sqrt{67.5}$ $\sqrt{64} < \sqrt{67.5}$ $\sqrt{81}$ — 8, 9
23. $\sqrt{81.49}$
24. $\sqrt{121.86}$ $\sqrt{121} \leftarrow \sqrt{121.86}$ $\sqrt{144}$ — 11, 12

Find the approximate side length of each square figure to the nearest whole unit.

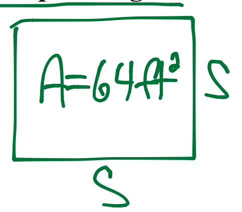
25. a rug with an area of 64 ft^2

8 ft

26. an exercise mat that is 6.25 m^2

27. a plate that is $\sqrt{49}$ cm^2

7cm



$A_{square} = S^2$

$A = 8ft \cdot 8ft = 64\ ft^2$

$64 = S^2$

$\sqrt{64} = S$

$8 = S$

Name______________________________________ Class_______________ Date_______________

1-3 Practice (continued)

Name the subset(s) of the real numbers to which each number belongs.

28. $\frac{12}{18}$ rational

29. -5

30. π irrational

31. $\sqrt{2}$

32. 5564 rational integer whole natural

33. $\sqrt{13}$

34. $-\frac{4}{3}$ rational

35. $\sqrt{61}$

Compare the numbers in each exercise using an inequality symbol.

36. $\sqrt{25}, \sqrt{64}$

37. $\frac{4}{5}, \sqrt{1.3}$

38. $\pi, \frac{19}{6}$

39. $\sqrt{81}, -\sqrt{121}$

40. $\frac{27}{17}$, 1.7781356

41. $-\frac{14}{15}, \sqrt{0.8711}$

Order the numbers from least to greatest.

42. $1.875, \sqrt{64}, -\sqrt{121}$

43. $\sqrt{0.8711}, \frac{4}{5}, \sqrt{1.3}$

44. $8.775, \sqrt{67.4698}, \frac{64.56}{8.477}$

45. $-\frac{14}{15}, 5.587, \sqrt{81}$

46. $\frac{100}{22}, \sqrt{25}, \frac{27}{17}$

47. $\pi, \sqrt{10.5625}, -\frac{15}{5.8}$

48. Marsha, Josh, and Tyler are comparing how fast they can type. Marsha types 125 words in 7.5 minutes. Josh types 65 words in 3 minutes. Tyler types 400 words in 28 minutes. Order the students according to who can type the fastest.